CO-ORDINATE GEOMETRY

1 To change from Cartesian coordinates to polar coordinates, for X write

$r \cos\theta$ and for y write $r \sin\theta$.

2 To change from polar coordinates to cartesian coordinates, for r^2 write

$X^2 + y^2$; for $r\cos\theta$ write X, for $r\sin\theta$

. Write y and for $\tan\theta$ write $\frac{y}{x}$.

3 Distance between two points (X_1, Y_1) and (X_2, Y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

4 Distance of (x_1, y_1) from the origin is $\sqrt{x_1^2 + y_1^2}$

5 Distance between (r_1, θ_1) and (r_2, θ_2) is

$$\sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)}$$

6 Coordinates of the point which divides the line joining (X_1, Y_1) and

(X_2, Y_2) internally in the ratio $m_1 : m_2$ are :-

$$\left\{\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \middle| \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right\}, (m_1 + m_2 \neq 0)$$

7. Coordinates of the point which divides the line joining (X_1, Y_1) and

(X_2, Y_2) externally in the ratio $m_1 : m_2$ are :-

$$\left\{\frac{m_1 X_2 - m_2 X_1}{m_1 - m_2} \middle| \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}\right\}, (m_1 - m_2 \neq 0)$$

8. Coordinates of the mid-point (point which bisects) of the seg. Joining (X_1, y_1) and $(X_2\, y_2)$ are :

$$\left[\frac{x_1 + x_2}{2} \quad , \quad \frac{y_1 + y_2}{2}\right]$$

9. (a) **Centriod** is the point of intersection of the medians of triangle.

(b) **In-centre** is the point of intersection of the bisectors of the angles of the triangle.

(c) **Circumcentre** is the point of intersection of the right (perpendicular) bisectors of the sides of a triangle.

(d) **Orthocentre is** the point of intersection of the altitudes (perpendicular drawn from the vertex on the opposite sides) of a triangle.

10.Coordinates of the centriod of the triangle whose vertices are (x_1 , y_1) ; (x_2 , y_2) ; (x_3 , y_3) are

$$\left\langle\frac{x_1 + x_2 + x_3}{3} \middle| \frac{y_1 + y_2 + y_3}{3}\right\rangle$$

11. Coordinates of the in-centre of the triangle whose vertices are A (x_1 ,y_1) ; B $(x_2 ,y_2 ,)$; C (x_3 ,y_3) and 1 (BC) = a, 1 (CA) = b, 1 (AB) = c.

are $\left\{\frac{ax_1 + bx_2 + cx_3}{a+b+c} \middle| \frac{ay_1 + by_2 + cy_3}{a+b+c}\right\}$.

12 Slope of line joining two points (x_1 ,y_1) and $(x_2 ,y_2)$is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

13. Slope of a line is the tangent ratio of the angle which the line makes with the positive direction of the x-axis. i.e. $m = \tan\theta$

14. Slope of the perpendicular to x-axis (parallel to y –axis) does not exist, and the slope of line parallel to x-axis is zero.

15. **Intercepts:** If a line cuts the x-axis at A and y-axis at B then OA is Called intercept on x-axis and denoted by "a" and OB is called intercept on y-axis and denoted by "b".

16. X= a is equation of line parallel to y-axis and passing through (a, b) and y = b is the equation of the line parallel to x-axis and passing through (a, b).

17. X= 0 is the equation of y-axis and y = 0 is the equation of x-axis.

18. Y = mx is the equation of the line through the origin and whose slope is m.

19. Y= mx +c is the equation of line in **<u>slope intercept</u>** form.

20. $\frac{X}{a} + \frac{Y}{b} = 1$ is the equation of line in the **<u>Double intercepts</u>** form, where "a" is x-intercept and "b" is y-intercept**.**

21. X cos a + y sin a = p is the equation of line in **<u>normal</u>** form, where "p" is the length of perpendicular from the origin on the line and α is the angle which the perpendicular (normal) makes with the positive direction of x-axis.

22. $Y - Y_1 = m (x - x_1)$ is the **<u>slope point form</u>** of line which passes through $(x_1, y_1)$and whose slope is m.

23. **<u>Two points form: -</u>** $y-y_1 = \frac{Y_2-y_1}{x_2-x_1}(x - x_1)$ is the equation of line which Passes through the points (x_1, y_1) and (x_2, y_2).

24. **<u>Parametric form</u>** :- $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ is the equation of line which

passes through the point (x_1 y_1)makes an angle θ with the axis and r is the distance of any point (x, y) from (x_1, y_1).

25. Every first degree equation in x and y always represents a straight line

ax + by + c = 0 is the general equation of line whose.

(a) Slope = $-\frac{a}{b}$ = - $\left[\frac{\text{coefficient of } x}{\text{cofficient of } y}\right]$

(b) X - intercept = $-\frac{c}{a}$

(c) Y- intercept = $-\frac{c}{b}$

26. Length of the perpendicular from (x_1, y_1) on the line

ax + by + c = 0 is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

27. To find the coordinates of point of intersection of two curves or two lines, solve their equation simultaneously.

28. The equation of any line through the point of intersection of two given lines is

(L.H.S. of one line) +K (L.H.S. of 2nd line) = 0

(Right Hand Side of both lines being zero)

TRIGONOMETRY

29. $SIN^2\theta + Cos^2\theta = 1$; $Sin^2\theta = 1 - Cos^2\theta$,

$Cos^2\theta = 1 - Sin^2\theta$

30. $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$;

$Cosec\,\theta = \frac{1}{\sin\theta}$; $\cot\theta = \frac{1}{\tan\theta}$

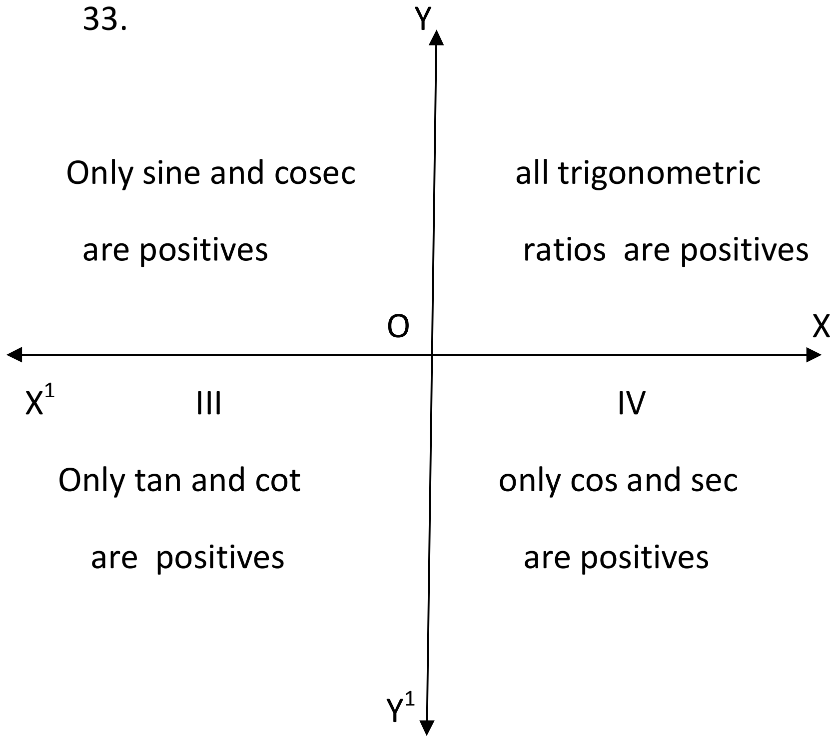
31. $1 + tan^2\theta = sec^2\theta$; $tan^2\theta = sec^2\theta - 1$;

$Sec^2\theta - tan^2\theta = 1$

32. $1 + cot^2\theta = cosec^2$; $cot^2\theta = cosec^2\theta - 1$;

$Cosec^2\theta - cot^2\theta = 1$

33.



34.

angle / ratio	0^0 O	30^0 $\frac{\pi}{6}$	45^0 $\frac{\pi}{4}$	60^0 $\frac{\pi}{3}$	90^0 $\frac{\pi}{2}$	120^0 $\frac{2\pi}{3}$	135^0 $\frac{3\pi}{4}$	150^0 $\frac{5\pi}{6}$	180^0 π
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\sqrt{\frac{3}{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

35. Sin ($-\theta$) = - Sin θ; cos ($-\theta$) = cos θ ; tan ($-\theta$) = - tan θ .

36.

sin (90 – θ) $=$ cos θ	sin (90 + θ) $=$ cos θ	sin (180 – θ) $=$ sin θ
cos (90 – θ) $=$ sin θ	cos (90 +θ) $= -$sin θ	cos (180 – θ) $= -$ cos θ
tan (90 – θ) $=$ cot θ	tan (90 +θ) $= -$cot θ	tan (180 – θ) $= -$ tan θ
cot (90 – θ) $=$ tan θ	cot (90+ θ) $= -$ tan θ	cot (180 – θ) $= -$ cot θ
sec (90 – θ) = cosec θ	sec (90 +θ) $= -$ cosecθ	sec (180 – θ) $= -$ secθ
cosec (90 – θ) =sec θ	cosec (90 +θ) = sec θ	cosec (180 – θ) = cosecθ

37. Sin (A + B) = SinA CosB + CosA SinB

Sin (A - B) = CosA SinB - SinA CosB

Cos (A + B) = CosA CosB - SinA CosB

Cos (A – B) = CosA CosB + SinA SinB

tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A \tan B}$

tan (A - B) = $\frac{\tan A-\tan B}{1+\tan A \tan B}$

38. tan $\left[\frac{\pi}{4}-A\right]=\frac{1-\tan A}{1+\tan A}$

tan$\left[\frac{\pi}{4}+A\right]=\frac{1+\tan A}{1-\tan A}$

39. SinC + SinD = 2 sin $\left(\frac{C+D}{2}\right)$ cos $\left(\frac{C-D}{2}\right)$

SinC - SinD = 2 cos $\left(\frac{C+D}{2}\right)$ sin $\left(\frac{C-D}{2}\right)$

CosC + CosD = 2 cos $\left(\frac{C+D}{2}\right)$ cos $\left(\frac{C-D}{2}\right)$

CosC - CosD = 2 sin $\left(\frac{C+D}{2}\right)$ sin $\left(\frac{D-C}{2}\right)$

40. 2 sin A cos B = sin (A + B) + sin (A-B)

2 cos A sin B = sin (A + B) - sin (A-B)

2 cos A COS B = cos (A +B) + cos (A-B)

2 sin A sin B = cos (A-B) - cos (A + B)

41. Cos (A +B). cos (A - B) = $\cos^2 A - \sin^2 B$

Sin (A +B). sin (A – B) = $\sin^2 A - \sin^2 B$

42. Sin 2θ = 2 sinθ cosθ = $\frac{2\tan\theta}{1+\tan^2\theta}$

43. Cos2 θ =$\cos^2\theta$ - $\sin^2\theta$ = $2\cos^2$ θ -1 = 1 – 2 $\sin^2$ θ

= $\frac{1 - \tan^2\theta}{1 + \tan^2\theta}$;

44. 1 + cos 2θ = 2 $\cos^2$ θ; 1 – cos 2 θ = 2 $\sin^2$ θ

45. tan 2 θ =$\frac{2\tan\theta}{1-\tan^2\theta}$;

46. sin 3 θ = 3 sin θ - 4 $\sin^3\theta$;

cos 3 θ = 4 $\cos^3\theta$ - 3 cos θ;

tan 3 θ = $\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

47. $\frac{a}{\sin A}$ = $\frac{b}{\sin B}$ =$\frac{c}{\sin C}$

48. Cos A = $\frac{b^2+c^2-a^2}{2\,bc}$; Cos B= $\frac{c^2+a^2-b^2}{2\,ca}$;

Cos C= $\frac{a^2+b^2-c^2}{2\,ab}$;

49. a = b cos C + c cos B; b = c cos A + a cos C ; c = a cos B + b cos A

50. Area of triangle =

$\frac{1}{2}$ bc sin A = $\frac{1}{2}$ ca sin B = $\frac{1}{2}$ ab sin c

51. $1 \pm \sin A = (\cos A/_2 \pm \sin A/_2)^2$

52. $\sec A \pm \tan A = \tan\left(\frac{\pi}{4} \pm A/2\right)$

53. Cosec A - cot A = tan $A/_2$

54. Cosec A + cot A = cot $A/_2$

PAIR OF LINES

1. A homogeneous equation is that equation in which sum of the powers of x and y is the same in each term.
2. If m_1 and m_2 be the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$, then
   $m_1 + m_2 + - \frac{2h}{b} = - \left(\frac{\text{coefficient of xy}}{\text{coefficient of } y^2}\right)$
   and $m_1 + m_2 = \frac{a}{b} = \frac{\text{coefficient of } x^2}{\text{coefficent of } y^2}$
3. If θ be the acute angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$, then
   $\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$
   These lines will be co –incident (parallel) if $h^2 = ab$ and perpendicular if a +b = 0.
4. The condition that the general equation of the second degree viz $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight line is
   $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

i.e. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$

5. $Ax^2 + 2hxy + by^2 = 0$ and $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are pairs of parallel lines.

6. The point of intersection of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is obtained by solving the equation $ax + hy + g = 0$ and $hx + by + f = 0$.

7. Joint equation of two lines can be obtained by multiplying the two equations of lines and equating to zero. (UV =0, where u = 0, v = 0).

8. If the origin is changed to (h,k) and the axis remain parallel to the original axis then for x and y put x' + h and y' + k respectively.

CIRCLE

1. $X^2 + y^2 = a^2$ is the equation of circle whose centre is (0, 0) and radius is a.

2. $(x – h)^2 + (y - k)^2 = a^2$ is the equation of a circle whose centre is (h, k) and radius is a.

3. $X^2 + y^2 + 2gx + 2fy + c = 0$ is a general equation of circle, its centre is (-g ,-f) and radius is $\sqrt{g^2 + f^2 - c}$.
4. Diameter form: - $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ is the equation of a circle whose (x_1, y_1) and (x_2 , y_2) are ends of a diameter.
5. Condition for an equation to represent a circle are :
   (a) Equation of the circle is of the second degree in x and y.

   (b) The coefficient of x^2 and y^2 must be equal.

   (c) There is no xy term in the equation (coefficient of xy must be zero).

1. To find the equation of the tangent at (x_1 , y_1) on any curve rule is:

   In the given equation of the curve for x^2 put xx_1; for y^2 put yy_1;

   for 2x put $x + x_1$ and for 2y put $y + y_1$

2. For the equation of tangent from a point outside the circle or given slope or parallel to a given line or perpendicular to a given line use $y = mx + c$ or $y - y_1 = m (x - x_1)$.
3. For the circle $x^2 + y^2 = a^2$
   (a) Equation of tangent at (x_1, y_1) is $xx_1 + yy_1 = a^2$

   (b) Equation of tangent at $(a \cos \theta, a \sin \theta)$ is $x \cos \theta + y \sin \theta = a$.

   (C) Tangent in terms of slope m is

$Y = mx \pm a\sqrt{m^2 + 1}$

4. For the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

   (a) Equation of tangent at (x_1, y_1) is

   $Xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

   (b) Length of tangent from (x_1, y_1) is

   $$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

10. For the point P (x, y) , x is abscissa of P and y is ordinate of P.

PARABOLA

1. Distance of any point P on the parabola from the focus S is always equal to perpendicular distance of P from the directrix i.e. SP = PM.

2. Parametric equation of parabola $y^2 = 4ax$ is $x = at^2$, $y = 2at$.

   Coordinates of any point (t) is $(at^2, 2at)$

3. Different types of standard parabola

Parabola	Focus	Directrix	Latus rectum	Axis of Parabola (axis of symmetry)

$Y^2 = 4ax$	(a, 0)	X = - a	4a	Y = 0
$Y^2 = -4ax$	(-a, 0)	X = a	4a	Y = 0
$X^2 = 4by$	(0, b)	Y = - b	4b	X = 0
$X^2 = -4by$	(0, -b)	Y = b	4b	X = 0

4. For the parabola $y^2 = 4ax$
   (a) Equation of tangent at (x_1, y_1) is
   $Yy_1 = 2a(x + x1)$.

   (b) Parametric equation of tangent at $(at_1^2, 2at_1)$ is

   $yt_1 = x + at_1^2$

   (c) Tangent in term of slope m is $y = mx + \frac{a}{m}$ and its point of contact is $(a/m^2, 2a/m)$

   (d) If P (t_1) and Q (t_2) are the ends of a focal chord then $t_2 t_1 = -1$

   (e) Focal distance of a point P (x_1, y_1) is $x_1 + a$.

E L L I P S E

Ellipse	Foci	Directrices	Latus Rectum	Equation of axis	Ends of L.R

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$	$(\pm ae, 0)$	$X = \pm \frac{a}{e}$ 1. Distance of any point on an ellipse from the focus = e (Perpendicular distance of the point from the corresponding Directrix) i.e. SP = e PM. 2. Different types of ellipse	$\frac{2b^2}{a}$	major axis Y = 0 minor axis x = 0	$(ae, \frac{b^2}{a})$ $(ae, \frac{-b^2}{a})$
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a < b)$	$(0, \pm be)$	$Y = \pm \frac{b}{e}$	$\frac{2a^2}{b}$	major axis x = 0 minor axis y = 0	$(\frac{a^2}{b}, be)$ $(\frac{-a^2}{b}, be)$

3 Parametric equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is $x = a \cos \theta$

and y = b sin θ .

4. For the ellipse $\frac{x^2}{y^2} + \frac{y^2}{b^2} = 1$, a> b, $b^2 = a^2 (1 = e^2)$

And $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a< b, $a^2 = b^2\ (1 - e)$

5. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b)

(a) Equation of tangent at x_1, y_1) is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1.$$

(b) Equation of tangent in terms of its slope m is

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

(c) Tangent at (a cos , b sin θ) is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

6. Focal distance of a point P (x_1 , y_1) is SP = $|a - ex_1|$

and SP = $|ex_1 + a|$

H Y P E R B O L A

1. Distance of a point on the hyperbola from the focus = e (Perpendicular distance of the point from the corresponding directrix) i.e. SP =ePM
2. Different types of Hyperbola

Hyperbola	Foci	Directrices	L.R	End of L.R	Eqn of axis
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$(\pm\ ae, 0)$	$X= \pm \frac{a}{e}$	$\frac{2b^2}{a}$	$(ae, \frac{b^2}{a})$ $(ae, -\frac{b^2}{a})$	Transverse axis y= 0 conjugate axis x = o
$\frac{y^2}{b^2} - \frac{x^2}{a^2} =1$	$(0, \pm\ be)$	$Y = \pm \frac{b}{e}$	$\frac{2a^2}{b}$	$(\frac{a^2}{b}, be)$ $(-\frac{a^2}{b}, be)$	Transverse axis x=0 conjugate axis y =0

3. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $b^2 = a^2 (e^2 - 1)$ and for

   $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, a2 = $b^2 (e^2 - 1)$.

4. Parametric equations of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are

   $X = a \sec \theta$, $y = b \tan \theta$

5. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

   (a) Equation of tangent at (x_1, y_1) are

   $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

   (b) Equation of tangent in terms of its slope m is

   $Y = mx \pm \sqrt{a^2m^2 - b^2}$

(c) Equation of tangent at (a sec, b tan θ) is

$$\frac{x \sec\theta}{a} - \frac{y \tan\theta}{b} = 1$$

(d) Focal distance of P (x_1, y_1) is S P = | $ex_1 - a$ | and S P = |$ex_1 + a$ |

SOLID GEOMETRY

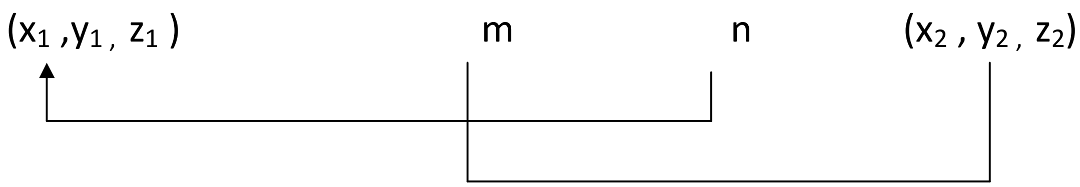
1. Distance between (x_1 , y_1 , z_1) and (x^2 , y^2, z^2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. Distance of (x_1 , y_1, z_1) from origin $\sqrt{x^21 + y^21 + z^21}$

3. Coordinates of point which divides the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) internally in the ratio m:n are

$$\left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right] \quad m + n \neq O$$



4. Coordinates of point which divides the joint of (x_1, y_1, z_1) and (x_2 ,y_2, z_2) externally in the ratio m:n are

$$\left[\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}, \frac{mz_2 - nz_1}{m - n}\right] \quad m - n \neq O$$

5. Coordinates of mid point of join of (x_1, y_1, z_1) and (x_2, y_2, z_2) are $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right]$.

6. Coordinates of centriod of triangle whose vertices are (x_1, y_1, z_1),

(x_2, y_2, z_2) and (x_3, y_3, z_3) are

$$\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

7. Direction cosines of x –axis are 1, 0, 0

8. Direction cosines of y –axis are 0, 1, 0

9. Direction cosines of z – axis are 0, 0, 1

10. If OP = r, and direction cosines of OP are l, m, n, then the coordinates of P are (l r, mr, nr)

11. If 1, m, n are direction cosines of a line then $l^2 + m^2 + n^2 = 1$

12. If l, m, n, are direction cosines and a ,b, c, are direction ratios of a line then $l = \frac{a}{\pm\sqrt{a^2+b^2+c^2}}$, $m = \frac{b}{\pm\sqrt{a^2+b^2+c^2}}$,

$n = \frac{c}{\pm\sqrt{a^2+b^2+c^2}}$,

13. If l , m, n, are direction cosines of a line then a unit vector along the line is $l\,\bar{i} + m\,\bar{j} + n\,\bar{k}$

14. If a, b, c are direction ratio of a line, then a vector along the line is $a\,\bar{i} + b\,\bar{j} + c\,\bar{k}$

VECTORS

1. $\bar{a} \cdot \bar{b} = ab\cos\theta = a_1 a_2 + b_1 b_2 + c_1 c_2.$

2. projection of $\bar{a}$ on $\bar{b}$ = $\frac{\bar{a} \cdot \bar{b}}{|\bar{b}|}$ and projection of b on a = $\frac{\bar{a} \cdot \bar{b}}{|a|}$

3. $\bar{a} \times \bar{b} = ab\sin\theta\,\hat{n}$ $\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

   $\bar{a} \times \bar{b} = -(\bar{b} \times \bar{a})$

4. $\bar{a} \cdot \bar{b} \times \bar{c} = [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

5. Vector area of Δ ABC is

$$\frac{1}{2}(\overline{AB} \times \overline{AC}) = \frac{1}{2}(\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a})$$

And area of Δ ABC = $\frac{1}{2}\ |\overline{AB} \times \overline{AC}|$

6. Volume of parallelepiped : $|\bar{a}\ \bar{b}\ \bar{c}|$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = |\overline{AB}\ \ \overline{AC}\ \ \overline{AD}|$$

7. Volume of Tetrahedram ABCD is = $\frac{1}{6}\ |\overline{AB}\ \ \overline{AC}\ \ \overline{AD}\ |$

8. Work done by a force $\vec{F}$ in moving a particle from A to B = $\overline{AB} \cdot \vec{F}$

9. Moment of force $\vec{F}$ acting at A about a point B is $\overline{M} = \overline{BA} \times \vec{F}$

P R O B A B I L T Y

1. Probability of an event A is P (A) $= \frac{n(A)}{n(S)}$ $0 \leq$ p () ≤ 1

2. p (AUB) = P (A) + P (B) - P (A∩B). IF A and B are mutually exclusive then P (A∩B) = 0 and P (A∪B) = P(A) + P(B)

3 P (A) = 1 – P (A) = 1 - P (A)

4. P(A∩B) = P(A) · P(B/A) = P(B) · P(A/B).

   IF A and B are independent events

   P(A ∩B) = P(A) · P(B)

5. P(A) = P(A∩B) + P(A∩B)

6. P(B) = P(A∩B) + P(A∩B)

7. $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = 1$; $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

$$\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = \lim_{\theta \to 0} \frac{\sin m\theta}{m\theta} \times m = m$$

$\lim_{\theta \to 0} \cos . = 1$; $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^n$

8 . $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$;

$$\lim_{x \to 0} (1 + kx)^{\frac{1}{X}} = \lim_{x \to 0} \left[(1 + kx)^{\frac{1}{KX}}\right]^k = e^K.$$

DIFFERENTIAL CALCULAS

1. F(x) = $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$; where f ' (x) is derivative of function f (x) with respect to x.

F (a) = $\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$

2. $\frac{d}{dx}$(a) = 0, where a is constant ; $\frac{d}{dx}$ (x) = 1,

$\frac{d}{dx}$ (ax) = a, $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$; $\frac{d}{dx}\left(\frac{1}{u}\right) = \frac{-1}{u^2} \times \frac{du}{dx}$

$$\frac{d}{dx}\left(\frac{1}{U^n}\right) = \frac{-n}{U^{n+1}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}} ; \frac{d}{dx}\sqrt{u} = \frac{1}{2.\sqrt{u}} \times \frac{d}{dx} U.$$ Where u = f(x)

3. $\frac{d}{dx}[x^n] = n\,[x]^{n-1}$; $\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$; $\frac{dy^n}{dx} = ny^{n-1}\frac{dy}{dx}$

4. $\frac{d}{dx}\log x = \frac{1}{x}$; $\frac{d}{dx}(\log u) = \frac{1}{u} \times \frac{du}{dx}$

$\frac{d}{dx}\log_a x = \frac{1}{x \log a}$; $\frac{d}{dx}\log_a u = \frac{1}{u \log a} \times \frac{du}{dx}$

5. $\frac{d}{dx}[a^x] = a^x \log a$; $\frac{d}{dx}[a^u] = a^u \log a \times \frac{du}{dx}$

6. $\frac{d}{dx}[e^x] = e^x$; $\frac{d}{dx}[e^u] = e^u \times \frac{du}{dx}$

7. $\frac{d}{dx}[\sin x] = \cos x$; $\frac{d}{dx}[\sin u] = \cos u \times \frac{du}{dx}$, e. g.

$\frac{d}{dx}\sin(4x) = \cos 4x \times \frac{d}{dx}4x = \cos 4x \times 4 = 4\cos 4x$

8. $\frac{d}{dx}[\cos x] = -\sin x$; $\frac{d}{dx}[\cos u] = -\sin u \times \frac{du}{dx}$

9. $\frac{d}{dx}$ tan x = $\sec^2 x$; $\frac{d}{du}$ tan u = $\sec^2 u \times \frac{du}{dx}$

10. $\frac{d}{dx}$ cot x = - $\text{cosec}^2 x$; $\frac{d}{dx}$ cot u = - $\text{cosec}^2 u \times \frac{du}{dx}$

11. $\frac{d}{dx}$ sec x = sec x tan x ; $\frac{d}{dx}$ sec u = sec u $\times$ tan u $\times \frac{du}{dx}$

12. $\frac{d}{dx}$ cosec x = - cosec x cot x ; $\frac{d}{dx}$ cosec u

= - cosec u $\times$ cot u $\times \frac{du}{dx}$

13. $\frac{d}{dx}\sin^2 x = 2 \sin x \frac{d}{dx}(\sin x) = 2 \sin x \cos x = \sin 2x$

$\frac{d}{dx}\sin^n x = n \sin^{n-1} \times \frac{d}{dx}\sin x = n \sin^{n-1} x \cos x$

14. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$; $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \times \frac{du}{dx}$

15. $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$; $\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}} \times \frac{du}{dx}$

16. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$; $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \times \frac{du}{dx}$

17. $\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$; $\frac{d}{dx}\cot^{-1}u = \frac{-1}{1+u^2} \times \frac{du}{dx}$

18. $\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$; $\frac{d}{dx}\sec^{-1}u = \frac{1}{u\sqrt{down\ u^2-1}} \times \frac{du}{dx}$

19. $\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$; $\frac{d}{dx}\operatorname{cosec}^{-1}u = \frac{-1}{u\sqrt{u^2-1}} \times \frac{du}{dx}$

20. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d}{dx}(uvw) = vw\frac{du}{dx} + uw\frac{dv}{dx} + uv\frac{dw}{dx}$

21. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, $v \neq 0$.

22. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

23. F (x + h) = f (x) + h f (x)

24. Error in y is $\delta y = \frac{dy}{dx} \times \delta x$, Relative error in

Y is $= \frac{\delta y}{y}$ and percentage error in y $= \times \frac{\delta y}{y} \times 100$

25. Velocity $= \frac{ds}{dt}$, acceleration $a = \frac{dv}{dt} = v\frac{dv}{ds} = \frac{d^2s}{dt^2}$

I T N T E G R A L   C A L C U L U S

1. $\int (u + v + w + \ldots)\ dx = \int u\ dx + \int v dx + \int w dx + \ldots$

2. $\int af(x) = a \int f(x)\ dx$, where ‘a’ is a constant.

3. $\int x^n\ dx = \frac{x^n+1}{n+1} + c$, $(n \neq -1)$;

$$\int (ax+b)^n = \frac{1}{a}\ \frac{(ax+b)^{n+1}}{n+1} + c$$

4. $\int [f(x)]^n\ f(x)\ dx = \frac{[f(x)^n + 1]}{n+1} + c$, $(n \neq -1)$

5. $\int \frac{1}{x}\ dx = \log x + c$;

$$\int \frac{1}{ax+b}\ dx = \frac{1}{a} \log [ax+b] + c\ ;$$

$$\int \frac{f'(x)}{f(x)}\ dx = \log\ |\ f(x)\ |\ + c\ ;$$

the integral of a function in which the numerator is the differential coefficient of the denominator is log (Denominator).

6. $\int \sqrt{x}\ dx = \frac{2}{3}\ x^{3/2} + c$;

$\int \sqrt{ax+b}\ dx = \frac{2}{3a}(ax+b)^{3/2} + c$

7. $\int a^x\ dx = \frac{a^x}{\log a} + c$;

$\int a^{bx+c}\ dx = \frac{1}{b}\frac{a^{bx+c}}{\log a} + c$

8. $\int e^x\ dx = e^x + c$; $\int e^{ax+b}\ dx = \frac{1}{a}e^{ax+b} + c$.

9. $\int \sin(ax+b)\ dx = \frac{-1}{a}\cos(ax+b) + c$;

$\int \sin x\ dx = -\cos x + c$

10. $\int \cos(ax+b)\ dx = \frac{1}{a}\sin(ax+b) + c$;

$\int \cos x\ dx = \sin x + c$

11. $\int \tan(ax+b)\ dx = \frac{1}{a}\log\sec(ax+b) + c$;

$\int \tan x\ dx = \log\sec x + c$

12. $\int \cot(ax+b)\ dx = \frac{1}{a}\log\sin(ax+b) + c$;

$\int \cot x\ dx = \log\sin x + c$

13. $\int \sec(ax+b)\ dx$

$= \frac{1}{a} \log \ | \sec (ax+ b) + \tan (ax + b) | + c$

$= \frac{1}{a} \log \tan \left|\frac{ax+b}{2} + \frac{\pi}{4}\right| + c$

$\int \sec x \, dx = \log |\sec x + \tan x| + c$

$= \log \tan \left(\frac{x}{2} + \frac{\pi}{4}\right) + c$

14. $\int \operatorname{cosec} (ax + b)dx$

$= \frac{1}{a} \log |\operatorname{cosec} (ax + b) - \cot(ax + b)| + c$

$= \frac{1}{a} \log \tan \left|\frac{ax+b}{2}\right| + c$

$\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + c$

$= \log \tan \left(\frac{x}{2} \right) + c$

15. $\int \sec^2 x \, dx = \tan x + c$;

$\int \sec^2(ax + b) \, dx = \frac{1}{a} \tan (ax + b) + c$

16. $\int \operatorname{cosec}^2 (ax +b) \, dx = \frac{-1}{a} \cot (ax +b) + c$;

$\int \operatorname{cosec}^2 x \, dx = - \cot$

17. $\int \sec(ax + b) \tan (ax +b) \, dx = \frac{1}{a} \sec (ax +b) + c$;

$\int \sec x \tan x \, dx = \sec x + c$

18. $\int \operatorname{cosec} (ax +b) \cot (ax +b) \, dx = \frac{1}{a} \operatorname{cosec} (ax +b) +c$;

$\int \operatorname{cosec} x \cot x \, dx = - \operatorname{cosec} x + c$

19. To integrate $\sin^2 x$, $\tan^2 x$, $\cot^2 x$ change to $\frac{1}{2}(1 - \cos 2x)$;

$\frac{1}{2}(1 - \cos 2x)$; $\frac{1}{2}(1 + \cos 2x)$; $\sec^2 x - 1$ and $\text{cosec}^2 x - 1$

Respectively

20. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$

21 $\int \frac{dx}{1+x^2} = \tan^{-1} x + c = -\cot^{-1} x + c$

22 $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \left(\frac{x}{a}\right) + c$;

$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\text{cosec}^{-1} x$

NINE IMPORTANT RESULTS

1. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c = -\cos^{-1} \left(\frac{x}{a}\right) + c$

2. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log \left[x + \sqrt{x^2 + a^2}\right] + c$

3. $\int \frac{dx}{x^2 - a^2} = \log \left[x + \sqrt{x^2 - a^2}\right] + c$

4. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a}\right) + c$

5. $\int \sqrt{x^2 + a^2}\ dx = \frac{x}{2}\ \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left|x + \sqrt{x^2 + a^2}\right| + c$

6. $\int \sqrt{x^2 - a^2}\ dx = \frac{x}{2}\ \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left[x + \sqrt{x^2 - a^2}\right] + c$

7. $\int \frac{dx}{a^2, -x^2} = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$

8. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

9. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + c$

INTEGRATION BY SUBSTITUTION

	If the integrand contain	Proper substitution to be used

1	$\sqrt{a^2 - x^2}$	X = a sin θ
2	$\sqrt{x^2 + a^2}$	X = a tan θ
3	$\sqrt{x^2 - a^2}$	X= a sec θ
4	$e^{f(x)}$	F(x) = t
5	Any odd power of sin x	Cos x = t
6	Any odd power of cos x	Sin x = t
7	Odd powers of both sin x and cos x	Put that function = t which is of the higher power.
8	Any inverse function	Inverse function = t
9	Any even power of sec x	Tan x = t
10	Any even power of cosec x	Cot x = t
11	Function of e^x	e^x = t
12	$\frac{1}{a+b\sin x}$, $\frac{1}{a+b\cos x}$, $$\frac{1}{a+b\cos x+c\sin x}$$	tan $\frac{x}{2}$ = t then dx = $\frac{2dt}{1+t^2}$ sin x = $\frac{2t}{1+t^2}$ cosx = $\frac{1-t^2}{1+t^2}$
13	$\frac{1}{a+b\sin 2x}$, $\frac{1}{a+b\cos 2x}$	tan x = t then dx = $\frac{dt}{1+t^2}$

		sin 2t = $\frac{2t}{1+t^2}$ cos 2x = $\frac{1-t^2}{1+t^2}$
14	$\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$	divide numerator and denominator by $\cos^2 x$ and put tan x = t
15	$\frac{1}{x(px^m + q)}$	$x^m = t$
16	Expression containing fractional power of x or (ax +b)	x or ax +b = t^k where k is the L.C.M of the denominators of the fractional indices.

INTEGRATION BY PARTS

1. Integral of the product of two function

= First function $\times$ Integral of 2^{nd} -

$\int$[differential coefficient of 1st $\times$ integral of 2nd] dx

i.e. $\int[\mathrm{I} \times \mathrm{II}]\,dx = \mathrm{I} \times \int \mathrm{II}\ dx - \int\left[\frac{d}{dx}\ \mathrm{I} \times \int \mathrm{II}dx\right] dx$

Note :

1. The choice of first and second function should be according to the order of the letters of the word **LIATE.** Where L = Logarithmic; I = Inverse; A = Algebric; T =Trignometric ; E = Exponential
2. If the integrand is product of same type of function take that function as second which is orally integrable.
3. If there is only one function whose integral is not known multiply it by one and take one as the 2nd function.

DEFINITE INTEGRALS

1. $\int_a^b f(x)\,dx = [g(x)]_a^b = g(b) - g(a)$, where $\int f(x)\,dx = g(x)$

2 $\int_a^b f(x)dx = \int_a^b f(t)\,dt = \int_a^b f(m)\,dm$

3 $\int_b^a f(x)\,dx = - \int_b^a f(x)\,dx$

4 $\int_a^b f(x) = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, $a < c < b$.

5 $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$; $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

6 $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if f is even

$\int_{-a}^{a} f(x)\,dx = 0$ if f is odd

7 $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

If $f(2a-x) = f(x)$ then $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$

e. g. $\int_0^{\pi} \sin^n x\,dx = 2\int_0^{\pi/2} \sin^n x\,dx$ as

$\sin^n x = \sin^n(\pi - x)$

NUMERICAL METHODS

1. **Simpson's Rule :** According to Simpson's rule the value $\int_a^b y\,dx$ is approximately given by $\int_a^b y\,dx$

$= \frac{h}{3} \left[y_0 + 4\left(y_1 + y_3 + y_5 \dots y_{n-1}\right) + 2\left(y_2 + y_4 + y_6 + \cdots y_{n-2}\right) + y_n \right]$

Where h = $\frac{b-a}{n}$, and y_0, y_1, y_2, y_3, -------- y_n are the values of y when x = a, a + h, a + 2h, -------, b

In words : $\int_a^b y \, dx = \frac{lenth\ of\ the\ sub\ interval}{3}$

X [$(sum\ of\ the\ Ist\ and\ last\ ordinate) + four\ (the\ sum\ of\ the\ remaining\ odd\ ordinates) + twice\ (the\ sum\ of\ all\ even\ ordinates\ )$]

2. **Trapezoidal rule :** According to Trapezoidal rule the value of $\int_a^b$ y dx is approximately given by $\int_a^b y \, dx$ $= \frac{h}{2} \left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + y_3 + \cdots y_{n-1}\right) \right]$

In words : $\int_a^b y \, dx = \frac{lenth\ of\ sub\ interval}{2}$

X [$sum\ of\ the\ first\ and\ last\ ordinates + two\ times\ remaining\ all\ ordinatrs$]

3. **Finite Differences :**

Δ f (a) = f (a + h)

Δ^2 f (a) = Δ f (a +h) - Δ f(a)

$\Delta^n \quad f(a) \quad = \quad \Delta^{n-1} f(a + h) - \Delta^{n-1} \quad f(a)$

$1 + \Delta = E$

$\Delta = E - 1$

$E \quad f(a) = f(a + h)$

$E^2 \; f(a) \quad = f(a + 2h)$

$E^n \quad f(a) = f(a + nh)$

In words : To obtain Δ of any function, for 'a' write a + h

In the function and subtract the function. If interval of

differencing is 1, than

$\Delta \quad f(a) = f(a + 1) - f(a)$

$\Delta^2 \; f(a) = \Delta \; f(a + 1) - \Delta \; f(a)$

4. Interpolation : Newton's Forward formula of interpolation.

$$t = \frac{x - x0}{h}$$

$$f(x_0 + th) = f(x_0) + t \; \Delta \; f(x_0) + \frac{t(t-1)}{2!} \; \Delta^2 \; f(x_0)$$

$$+ \frac{t(t-1)(t-2)}{3!} \; \Delta^3 \; f(x_0) + \_\_\_\_\_\_$$

$$Y = y_0 + t \, \Delta \; y_0 + \frac{t(t-1)}{2!} \; \Delta^2 \; y_0$$

$+ \quad \frac{t(t-1)\ (t-2)}{3!} \quad \Delta^3\ y_0\ +$ ______

Newton's Backward formula of Interpolation.

$t\ =\ \frac{x-xn}{h}$

$F(x_n\ +\ th)\ =\ f(x_n)\ \ +\ t\nabla\ f(x_n)\ +\ \frac{t(t+1)}{2!}\ \nabla^2\ f(x_n)$

$+\ \frac{t(t+1)(t+2)}{3!}\ \ \nabla^3\ f(x_n)\ +$ ______

or $y = y_n\ +\ t\nabla y_n\ \ +$

$\frac{t(t+1)}{2!}\ \nabla y_n + \frac{t(t+1)(t+2)}{3!}\ \nabla y_n\ +$

Bisection Method : If y = f(x) is an algebraic function and any a and b such that f (a) $>$ 0 and f (b) $<$ 0, then one root of the function f(x) = 0 lies between a and b , we take $c_1\ =\ \frac{a\ +\ b}{2}$ and check f (c_1)

If f (c_1) = 0, c_1 is the exact root if not and if f (c_1) $>$ 0, $\therefore$ f (c_1) . f (b) $<$ 0 $\therefore$ a root c_2 lies between c_1 and b. If not and if (c_1) $<$ 0, $\therefore$ f (c_1). f (a) $<$ 0, $\therefore$ a root c_2 lies between c_1 and a.

Keep on repeating till the desired accuracy of the root is reached.

False Position Method: If $y = f(x)$ is an algebraic function and for any x_0 and x_1 such that $f(x_0) > 0$ and $f(x_1) < 0$ have opposite signs, then a root of $f(x) = 0$ lies between x_0 and x_1

Let it be x_2

$$\therefore \; x_2 = x_1 - f(x_1) \; . \; \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)} \right]$$

Check $f(x_2)$ if $(fx_2) = 0$ then x_2 is exact root, if not and if $f(x_2) < 0$, $\therefore f(x_0) \, . \, f(x_2) < 0$, then a root x_3 lies between x_0 and x_2, then

$$X_3 = x_2 - f(x_2) \; . \; \left[\frac{x_2 - x_0}{f(x_2) - f(x_0)} \right]$$

Keep on repeating till the desired accuracy of the root is reached.

Newton – Raphson Method: The interactive formula in Newton - Raphson method is

$$X_{i+1} = x_i - \frac{f(x_i)}{f(x_i)} \quad , \; i \geq 1$$

Keep on repeating till the desired accuracy of the root is reached.

F O R C O M M E R C E

Lagrange's Interpolation formula : This is used when interval of differencing is not same.

If f(a), f(b), f(c), f(d), ______ bethe corresponding value of f(x) when x = a, b, c, d _______then

$$F(x) = \left[\frac{(x-b)(x-c)(x-d)\_\_\_\_\_\_}{(a-b)(a-c)(a-d)\_\_\_\_\_\_} \right] f(a)$$

$$+ \left[\frac{(x-a)(x-c)(x-d)\_\_\_\_\_\_}{(b-a)(b-c)(b-d)\_\_\_\_\_\_} \right] f(b)$$

$$+ \left[\frac{(x-a)(x-b)(x-d)\_\_\_\_\_\_}{(c-a)(c-b)(c-d)\_\_\_\_\_\_} \right] f(c)$$

$$+ \left[\frac{(x-a)(x-b)(x-c)\_\_\_\_\_\_}{(d-a)(d-b)(d-c)\_\_\_\_\_\_} \right] f(d)$$

$$+ \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

6 Difference Equations

Let the equation be $\emptyset(E)\ y_n = f(n)$

The complete solution = complimentary function (C.F.) +Particular Integral (P.I.)

When R.H.S. is zero , then only C.F. is required

Method to find C.F.

(1) Write the given equation in E.

(2) Form the auxiliary equation. This is obtained by equating to zero the coefficient of $y_{n.}$

(3) Solve the auxiliary equation. Following are the different cases

Case (1) If all the roots of the auxiliary equation are real and different. Let the roots be $m_{1,}$ m_2, m_3, then C.F. is (solution is)

$$Y_n = C_1 (m_1)^x + C_2 (m_2)^x + C_3 (m_3)^x$$

Case (ii) (1) Let two roots be real and equal, suppose the roots are m_1 and m_1 then general solution is

$$Y_n = (C_1 + C_2 x) (m_1)^x$$

(2) If three roots be equal and real suppose the roots are m_1, m_1, m_1, Then the general solution is

$$Y_n = (C_1 + C_2x + C_3x^2) (m_1)^x$$

Case (iii) One pair of complex roots.

Let the roots be $\alpha \pm \beta\, i$ where I = $\sqrt{-1}$ then the general solution is

$$Y_n = r^n\,(C_1 \cos n\theta + C_2 \sin n\theta)$$

where $r = \sqrt{a^2 + \beta^2}$, $\theta = \tan^{-1}({}^{\beta}/_{x})$

Statistics :

(I) Arithmeic mean or simply mean is denoted by $\bar{x}$

I.e. $\bar{x}$ is the mean of the x's

(II) Methods for finding the arithmetic mean for individual items.

(a) $\bar{x} = \frac{\sum xi}{n}$

(b) $\bar{x} = a + \frac{\sum Di}{n}$

Where a is assumed mean and $D_i = x_i - a$

(c) $\bar{x} = a + \left(\frac{\sum Di}{n}\right) I$

Where $D_i = \frac{x_i - a}{I}$

I is the length of class interval.

(2) **Methods for finding the arithmetic Mean for frequency distribution.**

(a) Direct Method

$$\bar{x} = \frac{\Sigma\ f_i\ x_i}{\Sigma\ f_i}$$

(B) Method of assumed mean

$$\bar{x} = a + \frac{\Sigma\ f_i\ D_i}{\Sigma\ f_i}$$

Where $D_i = x_i - a$

(C) Step deviation method, shift of origin method.

$$\bar{x} = a + \left(\frac{\Sigma\ f_i\ D_i}{\Sigma\ f_i}\right) h$$

Where $D_i = \frac{x_i - a}{h}$, and h is length of class interval.

(II) **Median -** If the variates are arranged in accending or descending order of magnitude, the middle value is called the median.

If there are two middle values then the mean of the variate is median.

Method of finding Median for a Group data –

Find the cumulative frequencies. Find the median group. Median group is the group corresponding to

$\frac{1}{2}$ (n + 1)th frequency.

The formula for the median is

Median = l + $\left(\frac{{}^{n}/_{2} - cf}{f}\right)$. I where l is the lower limit of median group.. i is the length of class interval f is the frequency of median group Cf is the cumulative frequency preceeding the median class.

(iii) Standard deviation (σ)

(a) S.D. $= \sigma = \sqrt{\frac{\sum (xi - \bar{x})^2}{n}} = \sqrt{\frac{\sum di^2}{n}}$

Where di $=$ xi $-$ $\bar{x}$

(b) Assumed mean method

S.D. $= \sigma = \sqrt{\frac{\sum Di^2}{n} - \left(\frac{\sum Di}{n}\right)^2}$

Where Di $=$ xi $-$ a , and a is assumed mean.

(c) S.D. $= \sigma = \sqrt{\frac{\sum xi^2}{n} - \left(\frac{\sum xi}{n}\right)^2}$

When the variates are small numbers.

For Grouped Data :

(a) Directed method σ = S.D. = $\sqrt{\frac{\sum \ fi \ xi^2}{\sum \ fi} - \left(\frac{\sum \ fi \ xi}{\sum \ fi}\right)^2}$

$= \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum \ fx}{N}\right)^2}$

Where $\sum \ fi = N$

(b) Method of assumed mean

S.D. = σ = $\sqrt{\frac{\sum \ fidi^2}{N} - \left(\frac{\sum \ fidi}{N}\right)}^2$

Where $D_1 = x_1 = a$, a is assumed mean.

(c) Step deviation or shift of origin method

σ = S.D. = i $\sqrt{\frac{\sum \ fD_i^{12}}{N} - \left(\frac{\sum \ fD_i}{N}\right)^2}$

Where $D_i = \frac{xi-a}{i}$, i is length of class interval.

Correlation and Regression .

(1) Coefficient of Correlation or Karl Pearson's coefficient of correlation.

$$r = \frac{\sum(x-\overline{x})\quad(y-\overline{y})}{\sqrt{\sum(x-\bar{x})^2 \quad \sum(y-\bar{y})^2}} = \frac{\sum \ d1d2}{\sqrt{\left[\sum {}^2_1\right] \quad \left[\sum {}^2_2\right]}}$$

where $d_1 = x - \bar{x}$ and $d2 = y - \bar{y}$

this is used when $\bar{x}$ and $\bar{y}$ are integers

(2) Correlation coefficientis independent of the origin of reference and unit of measurement if

$$U = \frac{x-a}{h} \quad \& \quad V = \frac{y-b}{k}$$

Than $r_{xy} = r_{uv}$

$$\therefore \mathbf{r} = \frac{\sum xy - \frac{\sum x \ \sum y}{N}}{\sqrt{\left[\sum x^2 - \frac{\sum(x)^2}{N}\right] \left[\sum y^2 - \frac{\sum(y)^2}{N}\right]}}$$

For bi variate frequency table

$$r = \frac{\sum xy - \frac{(\sum fx)\,.\,(\sum fy)}{N}}{\sqrt{\sum FX^2 - \frac{(\sum fx)^2}{N}} \quad \sqrt{\sum fy^2 - \frac{(\sum fy^2)}{N}}}$$

$$= \frac{\sum \mathbf{UV} - \frac{\sum \mathbf{U} \sum \mathbf{V}}{\mathbf{N}}}{\sqrt{\left[\sum \mathbf{U^2} - \frac{(\sum \mathbf{U})^2}{\mathbf{N}}\right] \quad \left[\sum \mathbf{V^2} - \left(\frac{\sum \mathbf{V}}{\mathbf{N}}\right)^2\right]}}$$

Karl person coefficient of correlation can also be expressed as

$$r = \frac{\sum xy - n\,\bar{x}\,\bar{y}}{\sqrt{\sum x^2 - n\overline{x^2}} \quad \sqrt{\sum y^2 - n\overline{y^2}}}$$

If the correlation is perfect then r = 1, if the correlation is negative perfect, then r = - 1, if there is no correlation, then r = 0

-1 $\leq$ r $\geq$ 1, r lies between -1 & 1

Regression lines

(1) The equation of the line of regression of y on x is

$$Y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

i.e. $y - \bar{y} = b_{yx} (x - \bar{x})$ where $byx = \frac{\sigma y}{\sigma x}$

(2) The equation of line of regression of x and y is

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

i.e. $x - \bar{x} = b_{xy} (y - \bar{y})$ $bxy = \frac{\sigma x}{\sigma y}$

(3) $b_{yx} = r \frac{\sigma_y}{\sigma_x}$ is called regression coefficient of y and x

(4) $b_{xy} = r \frac{\sigma_x}{\sigma_y}$ is called regression coefficient of x and y

(5) $r = \pm \sqrt{byx \quad bxy}$

(6) In the case of line of regression of y on x, its slope and regression cofficient are equal

(7) The regression line of y on x is used to find the value of y when the value of x is given

(8) In case of line of regression of x on y , its regression cofficient is reciprocal of its slope

(9) The regression line of x on y is used to find the value of x when the value of y is given

(10) $(\overline{x}, \overline{y})$ is the point of intersection of two regression lines

(11) If the line is written in the form y = a + bx, then this is the line of regression of y on x

If the line is written in the form x = a + by, then this is the line of regression of x on y

If both the lines are written in the form
ax + by + c = 0, and nothing is mentioned, then take first equation as the equation of line of regression of y on x and second as the equation of line of regression of x on y

Error of prediction (a) y on x $\delta_{yx} = \sigma_y \sqrt{1 - r^2}$

(b) x on y $\delta_{xy} = \sigma_x \sqrt{1 - r^2}$